Q1 [Alexander and Sadiku, 2009, Q6.13]

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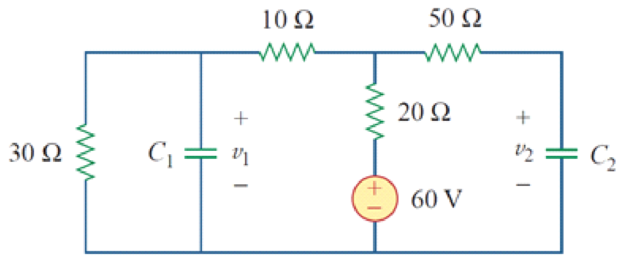
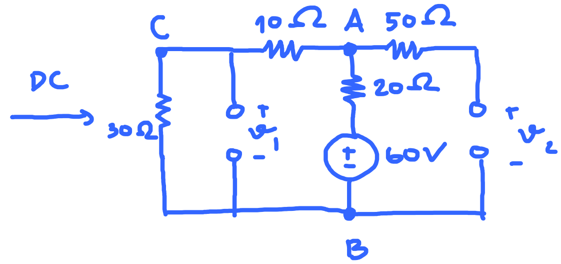
Under DC conditions, capacitors become open circuits.



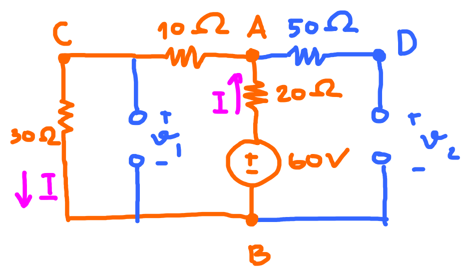


Note that the hanging branches created by the open circuits at the capacitor locations change the circuit to only one loop.

Method 1:

The current I in this loop is given by

$$\frac{60}{30+10+20} = \frac{60}{60} = 1\ A$$



Observe that $v_1 = v_C - v_B$ = the voltage across the 30Ω resistor

$= 30\Omega \times 1\ A = 30V$

$v_2 = v_D - v_B = v_A - v_B = 60 - I \times 20 = 60 - 20 = 40\ V$

no current through the 50Ω

⇒ Therefore, $v_A = v_D$.

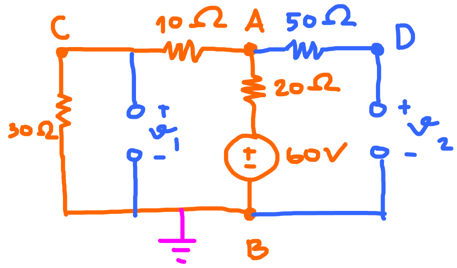
Method 2: Use voltage divider:

$$v_1 = \frac{30}{30+10+20} \times 60 = 30\ V$$

$$v_2 = \frac{10+30}{10+30+20} \times 60 = 40\ V$$

Method 3: Use nodal analysis:

At C:



At C:

$$\frac{V_C}{30} + \frac{V_C - V_A}{10} = 0 \Rightarrow V_C + 3V_C - 3V_A = 0 \Rightarrow V_C = \frac{3}{4} V_A$$

At D:

$$\frac{V_D - V_A}{50} + 0 = 0 \Rightarrow V_D = V_A$$

At A:

$$\frac{V_A - V_C}{10} + \frac{V_A - 60}{20} + \frac{V_A - V_D}{50} = 0 \Rightarrow 2V_A - 2V_C + V_A - 60 = 0$$

$$3V_A - 2V_C = 60$$

$$3V_A - \cancel{2}\frac{3}{\cancel{4}_{2}}V_A = 60 \Rightarrow \frac{3}{2}V_A = 60 \Rightarrow V_A = 40V$$

$$V_C = \frac{3}{4} \times 40 = 30V$$

$$v_1 = V_C = 30V$$

$$v_2 = V_D = V_A = 40V$$

Capacitor Combination

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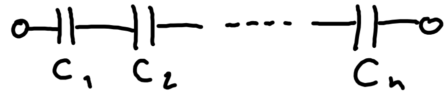
A couple of exercises in this HW deal with capacitor combination. To simplify our notation, we will use

$$\text{series}(C_1, C_2, \ldots, C_n)$$

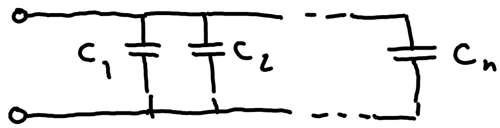
to mean the combined value of $C_1, C_2, \ldots, C_n$ when they are all connected in series:



We will use

$$\text{parallel}(C_1, C_2, \ldots, C_n)$$

to mean the combined value of $C_1, C_2, \ldots, C_n$ when they are all connected in parallel:



We know that

$$\text{series}(C_1, \ldots, C_n) = \left(\frac{1}{C_1} + \cdots + \frac{1}{C_n}\right)^{-1}$$

$$\text{parallel}(C_1, \ldots, C_n) = C_1 + \cdots + C_n$$

Note that we avoid using "//" to denote parallel combination for capacitors because it has been used for parallel combination of resistors which corresponds to different formula:

$$R_1 // R_2 = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Q2 [Alexander and Sadiku, 2009, Q6.19]

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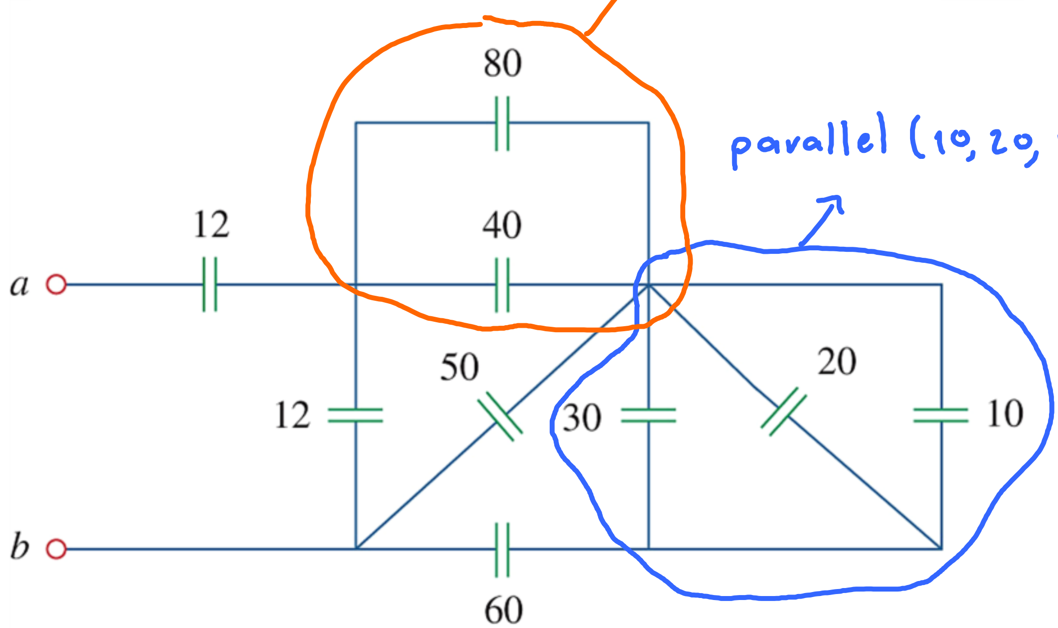
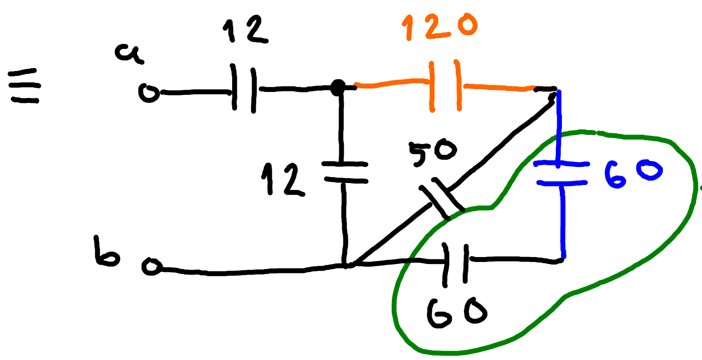
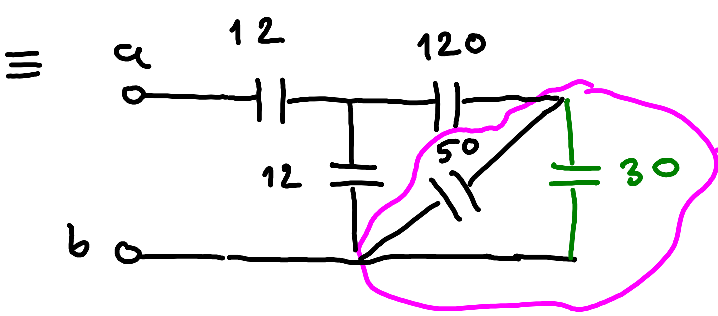
parallel (80, 40) = 120

parallel (10, 20, 30) = 60

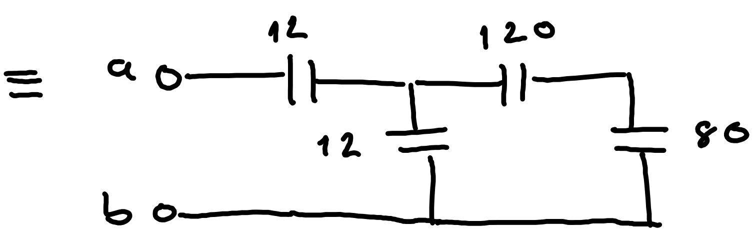




$\text{series}(60, 60) = \dfrac{1}{\frac{1}{60}+\frac{1}{60}} = \dfrac{60}{2} = 30$



parallel (30, 50) = 80



At this point, I'm too lazy to draw more pictures

At this point, I'm too lazy to draw more pictures. Our simplified combination above is easy enough to be represented in a one-line expression:

$$C_{ab} = \text{series}\left(12, \text{parallel}\left(12, \underbrace{\text{series}(120, 80)}_{\frac{120\times 80}{120+80} = \frac{\cancel{12}\cancel{0}\times 8\cancel{0}}{\cancel{2}\cancel{0}\cancel{0}} = 48}\right)\right)$$

$$48 + 12 = 60$$

$$\frac{60\times 12}{60+12} = \frac{\cancel{60}\times \cancel{12}}{\cancel{72}} = 10$$

$= 10\ \mu F$

don't forget the unit!!

Q3 [Alexander and Sadiku, 2009, Q6.20]

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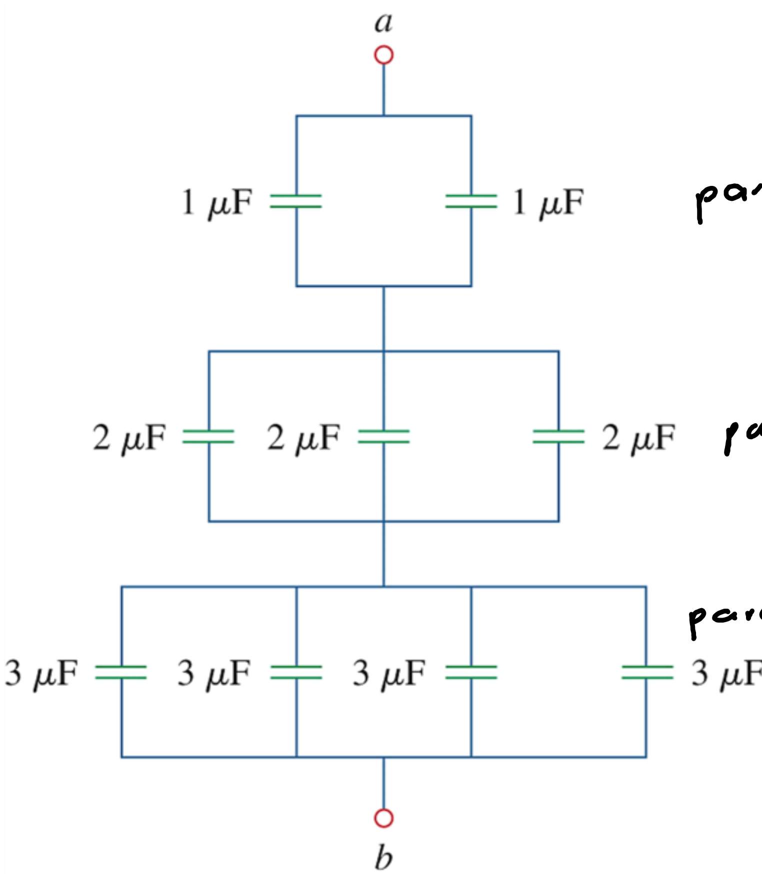
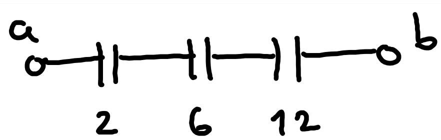


parallel (1, 1) = 2

parallel (2, 2, 2) = 6

parallel (3, 3, 3, 3) = 3 × 4
= 12

≡ 

$$C_{ab} = \frac{1}{\frac{1}{2} + \frac{1}{6} + \frac{1}{12}} = \frac{2}{1 + \underbrace{\frac{1}{3} + \frac{1}{6}}_{\frac{1}{2}}} = \frac{4}{3}\ \mu F$$

Q4 [Alexander and Sadiku, 2009, Q6.46]

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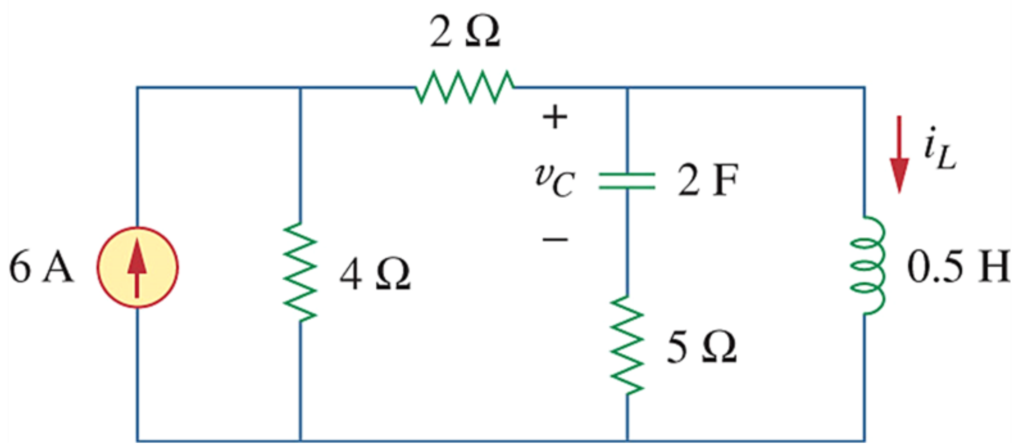
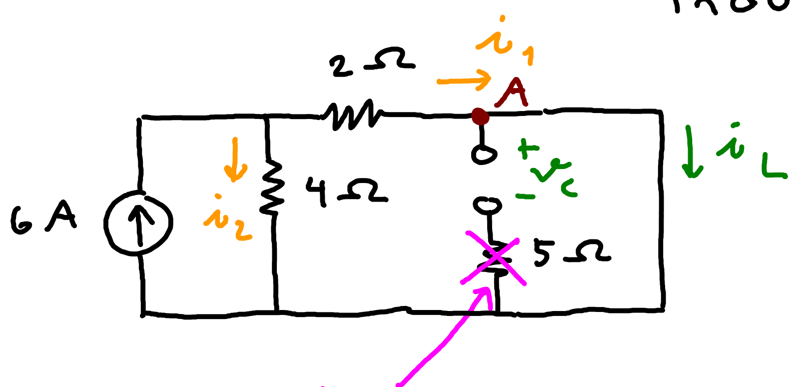


Under DC conditions, capacitor → open circuit
inductor → short circuit



No current through this resistor because of the open connection at the capacitor.

By current divider formula, $i_1 = 4\,A$
$i_2 = 2\,A$

$i_L = i_1 = 4\,A$

$v_c = v_A = 0\,V$ (There is a short connection from the top to the bottom of the circuit)

no current through the

through the
$5\,\Omega$ resistor

The energy stored in the capacitor is $w_c = \frac{1}{2} C \overset{0}{v_c^2} = 0\ \text{J}$

The energy stored in the inductor is $w_L = \frac{1}{2} L i_L^2$

$$= \frac{1}{2} \times \frac{1}{2} \times 4^2 = 4\ \text{J}$$

$v_c = 0\ \text{V}$ $\quad$ $w_c = 0\ \text{J}$

$i_L = 4\ \text{A}$ $\quad$ $w_L = 4\ \text{J}$

Q5 [Alexander and Sadiku, 2009, Q6.49]

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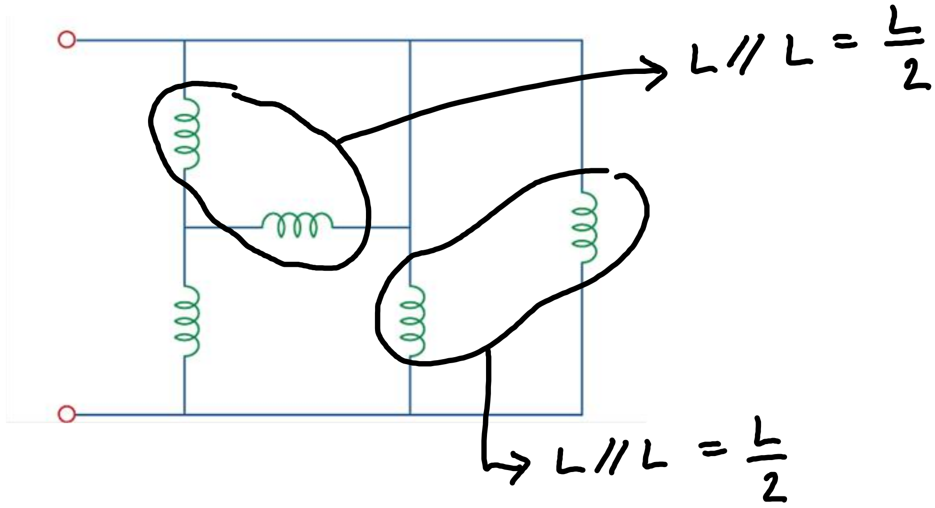

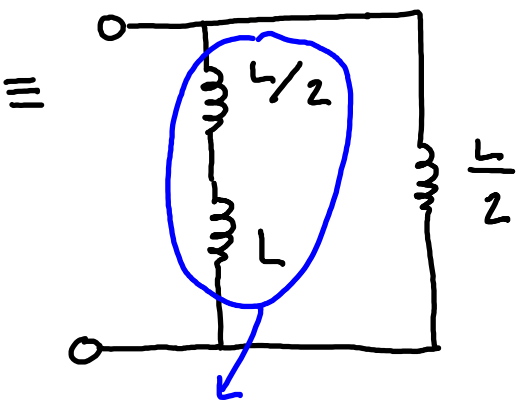

$$\frac{L}{2} + L = \frac{3L}{2}$$

$$L_{eq} = \frac{3L}{2} \,//\, \frac{L}{2}$$

$$= \frac{L}{2}\,(3 // 1)$$

$$= \frac{L}{2}\,\frac{3}{4} = \frac{3L}{8}$$

When $L = 10$,

$$L_{eq} = \frac{3 \times 10}{8} = 3.75 \text{ H}$$